Waiting-time distribution and market efficiency: evidence from statistical arbitrage

Sergey Osmekhin*and Frédéric Délèze†
Department of Finance and Statistics
Hanken School of Economics
P.O. Box 479, FI-00101, Helsinki, Finland

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Abstract

Statistical arbitrage strategies are market neutral trading strategies, which exploit market inefficiencies by taking long and short positions in a spread in order to generate a positive excess returns. This study analyses the relationship between the variation of waiting-time distribution of the traded spread and market efficiency. A statistical arbitrage optimisation procedure is obtained for both the Markovian and non-Markovian forms of Continuous-time random walk. A detrended spread between the two classes of ordinary shares of Royal Dutch Shell Plc and a spread between the two listings of Australia and New Zealand Banking Group Limited are used to observe market inefficient states. We find that the waiting-time and the survival probabilities have a significant impact on the price dynamics and indicate market inefficiency.

keywords:
Market inefficiency; Waiting-time distribution; Continuous-time random walk; Markovian process; non-Markovian process

1 Introduction

Market efficiency became a very prolific research topic since recent studies on pair trading [1, 2, 3, 4] have shown that predictable patterns inferred from historical data can be exploited to generate systematic excess returns. Pairs trading strategies (or statistical arbitrage) aim at taking a profit of the short-term price discrepancies observed on the spread between two assets sharing the same source of randomness. The spread between two co-integrated portfolios should be stationary and account for transaction costs and liquidity risk. The magnitude of the spread quantifies the level of mispricing. Statistical arbitrage strategies consist of optimising boundary conditions or barrier levels, under or above which, a long or short position in the spread is taken. The expected profit generated by the strategy on a given period depends on the barrier levels. They

*Corresponding author: sergey.osmekhin@hanken.fi
†frederik.deleze@hanken.fi

condition the expected frequency of the trades and the profit per trade: wide boundary conditions generate a large profit per trade but reduce the number of anticipated transactions. The number of trades per period is a renewal process and the average time between transactions is estimated by evaluating the first-passage time of the spread. Statistical arbitrage barrier levels provide a quantitative measure of short-term market efficiency: market is efficient within and inefficient outside the barriers. In this study we present a quantitative approach to measure the efficiency based on waiting time distribution.

In line with Osmekhin and Délèze [5, 6], we describe a pairs trading strategy, where the spread or log-price difference between two co-integrated portfolios is modelled with continuous-time random walk (CTRW). Continuous-time random walk is an extension of the classical random walk model initiated by Montroll and Weiss in 1965 [7]. CTRW was introduced as a theoretical approach to describe the diffusion process in solid state physics, where the waiting-time between two sequential space jumps of a moving particle is modelled stochastically. It models the dynamics of the probability density function of observing a particle in the space point x at time t. Similar processes take place in financial markets, where the time between transactions is stochastic and where trades induce price jumps [8]. Nowadays the CTRW framework is widely used in finance to predict and analyze the price behaviour of stock and derivatives [9, 10, 11] by calculating the probability density function (pdf) p of finding a certain price at a given time t.

Two main forms of CTRW has been recently described to model the price of financial assets [9, 10, 12]: a Markovian (memoryless) and a non-Markovian model. The CTRW Markovian equation describes the standard dynamic to model the price of financial instruments. The model can be seen as a generalisation of the geometric Brownian motion as it uses the asset return distribution as the unique driver to model the price fluctuation of an asset over time. Asset returns are known to be leptokurtic [13] and the assumption of independent and identically distributed equity returns underestimates the real probability of extreme events [14]. Fat-tailed distributions are supported by the general framework of CTRW.

The non-Markovian CTRW is an extension of the Markovian CTRW where both the time between transactions, called waiting-time, and the asset returns are modelled stochastically. The waiting-time distribution reflects the market liquidity. A transaction in a very illiquid market, i.e. when the waiting-time is abnormally long, translates into abrupt price changes while a transaction in a very liquid period has very little impact on price [15]. As the waiting-time distribution conveys relevant information about price formation we can expect the non-Markovian approach to outperform the memoryless model.

The master equation of the pdf p is a partial differential equation which includes the probability density functions of the waiting-time and asset returns. The solution of the master equation provides optimal barrier levels used in statistical arbitrage spread trading. The paper evaluates the level of mispricing using a fully stationary spread between the two listings of the Australia and New Zealand Banking Group Limited (ANZ) stocks. In practice, the spread between two cross-listed stocks are not necessarily perfectly stationary, even though both financial assets are traded on the same underlying product and, therefore, share the same source of information. We evaluate the profitability of the strategy using a detrended spread between Royal Dutch Shell A and B shares. Using tick-by-tick spread data we found that the distribution of the waiting-time or time between consecutive transactions varies with the level of mispricing for both spreads. The assumption of the exponential waiting-time distribution provides a quantitative measure of the market inefficiency and can be used as a market efficiency indicator.

Chapter 2 describes empirical data used in the study. Chapter 3 introduces the CTRW theoretical outlines and summarises the optimal trading strategy [5] and the two forms of CTRW used to model the dynamics of the spread [6]. The fourth chapter presents the results and discussion. We show the correlation between waiting-time distribution and market efficiency. Finally, chapter 5 concludes the article.

2 Empirical Data

To show the impact of the waiting-time distribution on market efficiency, two spreads between cross-listed stocks are constructed. The first spread is built around the two classes of ordinary shares of Royal Dutch Shell Plc: RDS.A and RDS.B[1]. The second spread is constructed with the two highly correlated listings of the Australia and New Zealand Banking Group Limited (ANZ) traded in Sydney on the Australian Stock Exchange (ANZ.AX) and in Wellington on the New-Zealand Stock Exchange (ANZ.NZ).

2.1 Royal Dutch Shell Spread

Both classes of Royal Dutch Shell ordinary shares have identical rights, except related to the dividend access mechanism, which only applies to the Class B ordinary shares. Class A ordinary shares have a Dutch source for tax purposes and are subject to Dutch withholding tax. Class B ordinary shares are entitled to a UK tax credit in respect of their proportional share of such dividends. Even though the information relative to Royal Dutch Shell Plc flows into both classes of shares, the log-price difference between the two classes

$$x_t = \log(RDS.A) - \log(RDS.B)$$

is not continuously stationary. The sample spread contains 3,187,942 mid-quote prices obtained from tickmarketdata.com [16]. Figure 1 displays the evolution of the spread x_t for the period between 1 March 2012 and 5 March 2013. After filtering out illiquid periods when the bid-ask spread exceeds 0.001, the spread series is detrended by applying a moving median of order 50,000. The detrended spread $\tilde{x}_t$ is illustrated on figure 2. The distribution of the detrended spread is presented in figure 3. Asset The number of lags of the moving median has been chosen to ensure that the process $\tilde{x}_t$ is stationary. Both the Augmented-Dickey

[1] http://www.shell.com/global/aboutshell/investor/share-price-information/difference-a-b.html

Fuller and Perron test reject the null hypothesis of unit root at 99% confidence level.

2.2 ANZ Spread

Unlike the spread formed of Royal Dutch Shell, the spread between ANZ.NX and ANZ.AX is continuously stationary at 99% confidence level and does not need to be detrended. ANZ.AX is traded in AUD on the Australian stock exchange in Sydney. The ANZ Bank New Zealand Limited, ANZ.NZ, is traded in NZD in Wellington on the New Zealand stock exchange. The time zone difference between the two exchanges is two hours. Using the Australian local time, the spread, x, is computed as follows:

$$x_t = \log(ANZ.AX) - \log(ANZ.NZ) + \log(FX_{AUD/NZD}) \tag{1}$$

ANZ.NZ is traded between 10:00 and 16:45 local time and ANZ.AX between 10:00 and 16:00 Sydney time. The spread depicted in figure 4 is computed for the period between 4 January 2012 and 8 March 2013 [16]. The distribution of the detrended spread is presented in figure 5. To prevent information asymmetry, the spread is calculated when both markets are open. It includes 306,071 observations.

3 Theory

The present section is divided into two parts. The first part describes the continuous-time random walk (CTRW) approach with Markovian and non-Markovian masters equations. The second part explains the optimisation of the statistical arbitrage trading strategy.

3.1 Continuous-Time Random Walk

Most studies in finance, including the market microstructure literature, sample asset prices at regular time intervals and only models prices as stochastic processes. However, the time between transactions provides predictive information regarding future asset prices and should be not neglected. This study uses the CTRW of Montroll and Weiss [7] to describe the price dynamics at a tick level, where both the price and time between two transactions are modelled with random variables. CTRW has already been applied in finance to measure the impact of option price on underlying price changes [11]. We follow the approach of Scalas *et al* [17, 10, 12] and introduce the following notation:

$x(t) = \log S(t)$	: logarithm of the asset price S at time t.
$\tau_i = t_{i+1} - t_i$	: waiting-time.
$\xi_i = x(t_{i+1}) - x(t_i)$	: log-return of asset price S at time t.
$\varphi(\xi, \tau)$	: joint prob. of returns and waiting-time.
$\psi(\tau) = \int_{-\infty}^{\infty} \varphi(\xi, \tau) d\xi$	: waiting-time pdf.
$\lambda(\xi) = \int_0^{\infty} \varphi(\xi, \tau) d\tau$	: asset return pdf.
$p(x, t)$	: pdf of finding x at time t.
$\hat{f}(\kappa) = \int_{-\infty}^{\infty} e^{i\kappa x} f(x) dx$	: Fourier transform of $f(x)$
$\tilde{f}(s) = \int_0^{\infty} e^{-st} f(t) dt$	: Laplace transform of $f(t)$.
$\widetilde{\hat{f}}(\kappa, s) = \int_0^{\infty} \int_{-\infty}^{\infty} e^{-st+i\kappa x} f(x, t) dx dt$	: Fourier-Laplace transform of $f(x, t)$

Given a transaction at time t_{i-1}, ψ represents the probability density function that a transaction take place at time $t_{i-1} + \tau$. Hence, the probability that a transaction is carried out within $\tau \leq t_i - t_{i-1} \leq \tau + d\tau$ is $\psi(\tau)d\tau$. So, the normalisation condition on the waiting-time is $\int \psi(\tau)d\tau = 1$.
λ represents the transaction probability density function that the log-price jumps from x to $x + \xi$. Its normalisation condition is $\int \lambda(\xi)d\xi = 1$. The normalisation condition on the joint probability density between waiting-time and returns is, therefore, $\int\int \varphi(\xi, \tau)d\xi d\tau = 1$. The probability that the log-price does not change during a period greater or equal to τ, also called survival probability until time instant t at the initial position $x_0 = 0$, denoted by $\Psi(\tau)$, is $\Psi(\tau) = 1 - \int_0^{\tau} \psi(t)dt = \int_{\tau}^{\infty} \psi(t)dt$.

Montroll and Weiss [7] show that the Fourier-Laplace transform of $p(\kappa, s)$ satisfies:

$$\widetilde{\hat{p}}(\kappa, s) = \frac{1 - \tilde{\psi}(s)}{s} \frac{1}{1 - \widetilde{\hat{\varphi}}(\kappa, s)} \tag{2}$$

Assuming that the time between two transactions and the returns are independent and that the time between transactions are i.i.d., the joint probability is the product of the return and the waiting-time probability density functions, i.e. $\varphi(\xi, \tau) = \lambda(\xi)\psi(\tau)$. Equation (2) can then be rewritten as:

$$\widetilde{\hat{p}}(\kappa, s) = \frac{1 - \tilde{\psi}(s)}{s} \frac{1}{1 - \hat{\lambda}(\kappa)\tilde{\psi}(s)} = \frac{\tilde{\Psi}(s)}{1 - \hat{\lambda}(\kappa)\tilde{\psi}(s)} \tag{3}$$

Reorganising the terms of (3) as $\widetilde{\hat{p}}(\kappa, s) = \tilde{\Psi}(s) + \tilde{\psi}(s)\hat{\lambda}(\kappa)\widetilde{\hat{p}}(\kappa, s)$ and inverting the Fourier-Laplace transform, we obtain the master equation of $p(x, t)$:

$$p(x, t) = \delta(x)\Psi(t) + \int_0^{\infty} dt' \psi(t - t') \int_{-\infty}^{\infty} \lambda(x - x') p(x', t') dx' \tag{4}$$

The initial condition is that the log-price is initially at its origin $x = 0$, i.e. $p(x, 0) = \delta(x)$ where $\delta(x)$ is the Dirac function.
The probability of finding log-price x at time t, $p(x, t)$, in the master equation (4) is the sum of two terms: an initial condition, i.e. the survival probability up to time instant t, $\Psi(t)$, times the jump pdf at point x, $\delta(x)$, and a spatio-temporal convolution term.

The second term is the time aggregation over the period $[0, t]$ of the marginal contribution to $p(x, t)$ of the log-price jump from $x' \in \mathbb{R}$ to x at time $t' < t$: $\int_{-\infty}^{\infty} \lambda(x - x')p(x', t')dx'$ by the probability waiting-time probability $\psi(t - t')dt'$. As described in [17], this form of the master equation shows the non-local and non-Markovian character of the CTRW:

$$\frac{\partial}{\partial t}p(x,t) + \int_0^t \phi(t - t')p(x,t')dt' = \int_0^t \phi(t - t') \int_{-\infty}^{\infty} \lambda(x - x')p(x', t')dx'dt' \quad (5)$$

where the kernel $\phi(t)$ is defined through its Laplace transform:

$$\tilde{\phi}(s) = \frac{s\tilde{\psi}(s)}{1 - \tilde{\psi}(s)} \quad (6)$$

An alternative form of the master equation (4) was proposed in Mainardi *et al* [10], which is the solution of the Green function or the fundamental solution of Cauchy problem with the initial condition $p(x, 0) = \delta(x)$:

$$\int_0^t \phi(t - t')\frac{\partial}{\partial t'}p(x,t')dt' = -p(x,t) + \int_{-\infty}^{\infty} \lambda(x - x')p(x', t)dx' \quad (7)$$

This form of the master equation is clearly non-Markovian as $\phi(s)$ is defined as a function of the survival probability. As $\tilde{\phi}(s) = 1$, i.e. $\phi(s) = \delta(x)$, the master equation for the CTRW becomes Markovian:

$$\frac{\partial}{\partial t}p(x,t) = -p(x,t) + \int_{-\infty}^{\infty} \lambda(x - x')p(x', t)dx' \quad (8)$$

with initial condition of $p(x, 0) = \delta(x)$.

Figure 6 illustrates the modelled solution of the Markovian master equation (8). At time $t = 0$, the probability density function $p(x, t)$ is a delta Dirac function because the current price is known. The uncertainty then increases with time and broadens $p(x, t)$. The skewness of the distribution $\lambda(x)$, modelled in the figure with an exponential distribution, orientates $p(x, t)$. The cross-sectional view at a given point in time $t > 0$ is the distribution of asset returns. Thus, $p(x, t = 200 \text{ sec})$ reflects the expected distribution of the log-price x 200 seconds ahead. The difference between the Markovian and non-Markovian approaches is illustrated in figure 7. The top three graphs show cross-sectional views of the Markovian, non-Markovian and its difference at time step $t = 4$. The bottom graph gives a three dimensional representation of the differences between the two PDEs over time. The non-Markovian approach is significantly different from the Markovian approach and should be used for the price behaviour analysis.

3.2 Statistical arbitrage trading strategy optimization

One of the key considerations in pairs trading consists of finding optimal barrier levels, which determine the entry and exit levels of the strategy and condition the frequency of the trades. The total time of the strategy is the sum of the expected time it takes for the spread to go from the entry level until the exit level and the time to go from the exit level back to the entry level, i.e. $\mathcal{T}_{total} = \mathcal{T}_{enter} + \mathcal{T}_{exit}$.

The trading frequency is therefore $\frac{1}{\mathcal{T}_{total}}$. We follow the same approach as Bertram [18] and compute the optimal boundary levels as a function of the first-passage time of the spread. The first-passage time of x_t is defined as the first time that the logarithm of the spread process x_t reaches an upper boundary b_1 or a lower boundary b_2:

$$\mathcal{T}_{[b_1,b_2]}(x_0) = \inf_{t_0 \leq t}\{t|b_2 < x_t < b_1; x_0 = x(0)\}$$

Assuming that the probability densities of finding a log-spread x at a future time t, denoted as $p(x,t|x_0,t_0)$, satisfy the absorbing initial conditions $p(x_L,t) = 0$ and $p(x_U,t) = 0$, $x_L < x_U$, the probability that the process x reaches the boundaries is:

$$G(t; x_U, x_L) = 1 - \int_{x_L}^{x_U} p(x,t|x_0,t_0)dx$$

The corresponding density function solves:

$$g(t|x_0,t_0) = \frac{\partial}{\partial t}G(t|x_0,t_0) = \int_{x_U}^{x_L} \frac{\partial}{\partial t}p(x,t|x_0,t_0)dx$$

Given two barrier levels b_1 and b_2, $b_1 < b_2$, the probability density function of the first-passage time $f(t;b_1,b_2)$ is the convolution of the density function g from the lower limit until the upper limit with the density function g from the lower boundary until the upper limit, i.e.

$$f(t;b_1,b_2) = g_{[-\infty,b_2]}(t;b_1,t_0) \otimes g_{[b_1,\infty]}(t;b_2,t_0) \tag{9}$$

The expected trading length solves $E[\mathcal{T}_{total}] = \int_0^\infty tf(t;b_1,b_2)dt$ and the expected trade frequency and variance:

$$\begin{aligned} E[\frac{1}{\mathcal{T}_{total}}] &= \int_0^\infty \frac{1}{t}f(t;b_1,b_2)dt \\ Var[\frac{1}{\mathcal{T}_{total}}] &= \int_0^\infty \frac{1}{t^2}f(t;b_1,b_2)dt - E[\frac{1}{\mathcal{T}_{total}}]^2 \end{aligned}$$

A trading strategy is optimal if the boundaries b_1 and b_2 maximise an objective function, which typically is the expected return of a portfolio μ_p or its Sharpe ratio. With fixed barriers b_1 and b_2 the return per trade is deterministic $b_2 - b_1 - c$ where c is the transaction cost, but the time between trades is stochastic and depends on the first-passage time of x_t.
The expected profit and variance per trade frequency are:

$$\begin{aligned} \mu_p &= (b_2 - b_1 - c)E[\frac{1}{\mathcal{T}_{total}}] \\ &= (b_2 - b_1 - c)\int_0^\infty \frac{1}{t}f(t;b_1,b_2)dt \qquad (10) \\ \sigma_p &= (b_2 - b_1 - c)^2 Var[\frac{1}{\mathcal{T}_{total}}] \\ &= (b_2 - b_1 - c)^2\int_0^\infty \frac{1}{t^2}f(t;b_1,b_2)dt - \mu_p^2 \qquad (11) \end{aligned}$$

Optimal barrier levels, defined as a function of the trading cost, is obtained by maximising the expected return or Sharpe ratio of the strategy. The market state beyond the barrier levels indicates market inefficiency. The profitability of the statistical arbitrage strategy is illustrated in figure 8 as a function of the transaction cost and barrier level.

4 Results and Discussion

4.1 Relation between waiting-time distribution and market efficiency

This section tests whether the waiting-time distribution depends on the state of the spread process. We expect a shorter time between transactions when the spread reaches the tail of its historical distribution.
Figure 9 shows the waiting-time distribution of spread between RDS.A and RDS.B. The left-side graph sketches the waiting-time distribution between 0 and 10 seconds. The first bar shows that more than half of the deals happens within a second. Excluding the first bar, the waiting-time distribution is well fitted with an exponential distribution. The right-side graph displays the long-life waiting-time distribution, where the time exceeds 10 seconds. A similar pattern is observed for the waiting-time distribution of ANZ spread in figure 10. Figures 11 and 12 present the waiting-time distribution within and outside barrier levels for RDS- and ANZ-spreads, respectively. The barrier levels are first set such that 90% of the observations lies within the boundaries. The blue bar reports the estimation of the waiting-time distribution using all points surrounded by the barriers. The red bar depicts the waiting-time distribution when only transactions outside of the barriers are taken into account. We can see that the market activity is higher, i.e. the time between truncation is shorter, when arbitrage opportunities occur and thus when the spread process is outside of its long-term mean.

Figure 11 shows that 52.6% of the RDS transactions are executed within one second when the spread stays within its 90% quantile while 52.8% of the transactions occur in less than one second when the spread is beyond its 10% extreme values. The same pattern is observed for ANZ spread as illustrated in figure 12. 34% of the ANZ transactions are executed within 5 seconds when the spread is within the boundaries while more than 35% of the transactions happens in the same time interval should the spread exceed its 10% extreme values. In the second case, the barrier levels are defined such as 99% of the spread prices stays within the boundaries. The green bar outlines the waiting-time distribution when the spread is abnormally high or low. A significant change in the waiting-time distribution is observed when the barrier levels are placed wider to encompass 99% instead of 90% of spread prices.

Table 2 summarises the results and reports the probability that trades occur within the next five seconds for the RDS spread and within 25 seconds for the less liquid ANZ spread for each of the three cases. The more extreme the spread prices, the shorter the time between transactions. This finding supports the efficient market hypothesis. Arbitrageurs place orders as soon as the spread abnormally diverges from its long-term equilibrium and, hence, shorten the time between transactions.

The waiting-time probability density function $\psi(\tau)$ is modelled as an exponential distribution of parameter θ

$$\begin{cases} \theta e^{-\theta\tau} & \text{if } \tau \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Modelling $\psi(\tau)$ as an exponential distribution simplifies the discretisation of the non-Markovian stochastic differential equation. Indeed, the Laplace transform of the kernel $\phi(t)$ is constant

$$\tilde{\phi}(t) = \frac{t\tilde{\psi}(t)}{1-\tilde{\psi}(t)} = \frac{t\frac{\theta}{t+\theta}}{1-\frac{\theta}{t+\theta}} = \theta \tag{12}$$

and, therefore, $\phi(t) = \theta\delta(t)$ where $\delta(t)$ is the Dirac function at t. The discretisation of the non-Markovian master equation (7) shows that $p(x, t+1)$, the probability density function of finding a log-price x at a future time $t+1$, is the sum of the survival probability up to time t times $p(x,t)$ with the solution of the Markovian stochastic density function (8):

$$\underbrace{p(x,t+1)}_{non-Markovian\ pdf} = \underbrace{(1-\frac{1}{\theta})}_{survival\ probability} \quad p(x,t) + \frac{1}{\theta} \quad \underbrace{p(x,t+1)}_{Markovian\ pdf\ (8)} \tag{13}$$

$$= \Psi p(x,t) + (1-\Psi)p(x,t+1) \tag{14}$$

where $\Psi = 1 - 1/\theta$ is the survival probability. Sudden and large bursts in market liquidity, mainly driven by macroeconomic news announcements [?], cause jumps in the survival probability. The effect of discontinuous survival probability on the asset distribution $p(x,t)$ is modelled in equation (13).
When modelled with an exponential distribution $\psi(\tau) = \theta e^{-\theta\tau}$ the optimal parameter θ for the waiting-time distribution of RDS is 0.482, 0.523 and 0.550 for the 90% inside, 10% outside and 1% outside distribution, respectively. The corresponding fitted value θ for ANZ spread is 0.0445, 0.0474 and 0.0555. The higher the parameter θ, the shorter the time between transactions. It defines whether the market is in an active or inactive phase and can be used as an indicator of market efficiency.

4.2 Non-Markovian trading strategy with detrended spread

The trading strategy optimises the barrier levels b_1 and b_2 to maximise the expected return μ_p. The probability density function $\lambda(\xi)$ of the detrended spread distribution of RDS is not normal as depicted in figure 3. The distribution exhibits fat tails in the region of $\pm(0.4-0.7\%)$. As such, the asset return distribution $\lambda(\xi)$ and waiting-time distribution $\psi(\tau)$ are inferred from historical spread prices $\tilde{x}_t$. The non-Markovian trading strategy (7) yields a profit of 26.0% p.a. with a trading cost of 0.05% for optimal barrier levels of $\pm 0.11\%$. Table 1 contains the results of the strategy for various barrier levels with a fixed cost of 0.05% per deal. Surprisingly, the optimal barrier levels remain the same whether the strategy is applied to the original spread x_t or the detrended process $\tilde{x}_t$. The algorithm is profitable for a range of barrier levels up to $\pm 0.4\%$. We would expect barrier levels to be wider for highly non-stationary processes than for detrended spreads. With the assumption of short non-stationary periods in the overall stationary spread process, we believe that the non-Markovian trading strategy (10-7) can equally be optimised with a detrended spread process.

5 Conclusion

In this article we analyse the relationship between the waiting-time distribution function of the traded spread and market efficiency. We show that inefficient price states outside of optimal barrier levels rapidly converge back to efficient price states within optimal boundaries. The farther the spread price diverges from its mean, the quicker is the mean-reversion. The barrier levels are obtained from a non-Markovian trading strategy using both a detrended spread between the two classes of ordinary shares of Royal Dutch Shell Plc and the two listings of Australia and New Zealand Banking Group Limited. We show that the parameter of the waiting-time exponential distribution is a good indicator of market efficiency. In addition, we analyse the optimal trading strategy for non-stationary process. Our results prompt further theoretical and empirical studies on the duration of inefficient price states.

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7 Figures and Tables

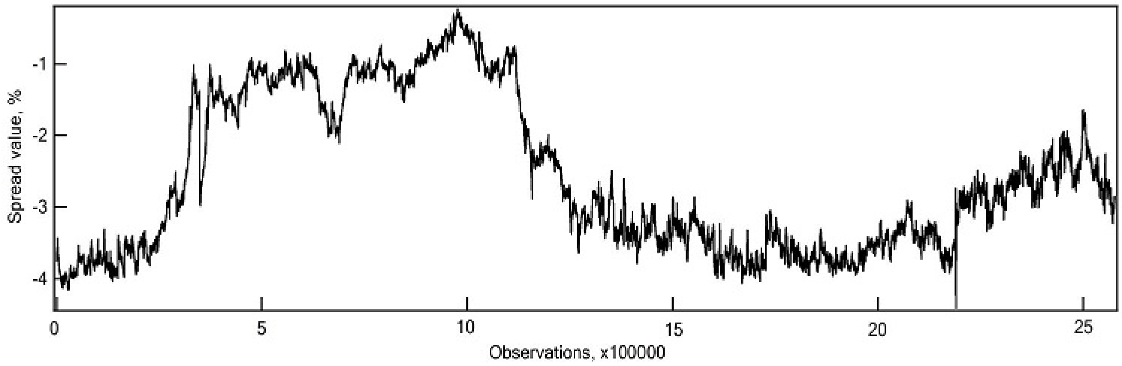

Figure 1: Log-price difference between RDS.A and RDS.B.

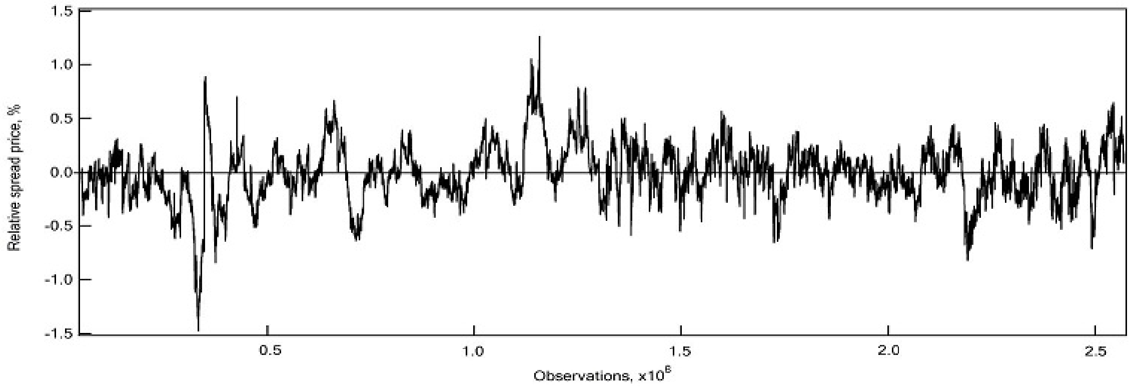

Figure 2: Detrended log-price difference between RDS.A and RDS.B.

Barrier	Profitability (with cost of 0.05% per deal)	Deals
0.05%	63.2% (14.1%)	982
0.08%	53.3% (24.6%)	574
0.11%	45.3% (26.0%)	386
0.15%	39.4% (25.8%)	272
0.20%	34.2% (25.0%)	184
0.30%	27.8% (22.2%)	112
0.40%	26.2% (22.3%)	78
0.50%	14.9% (12.9%)	40

Table 1: Using MA price with a 50000 averaging number gives 45.3% (26.0% with costs) profit (386 deals)

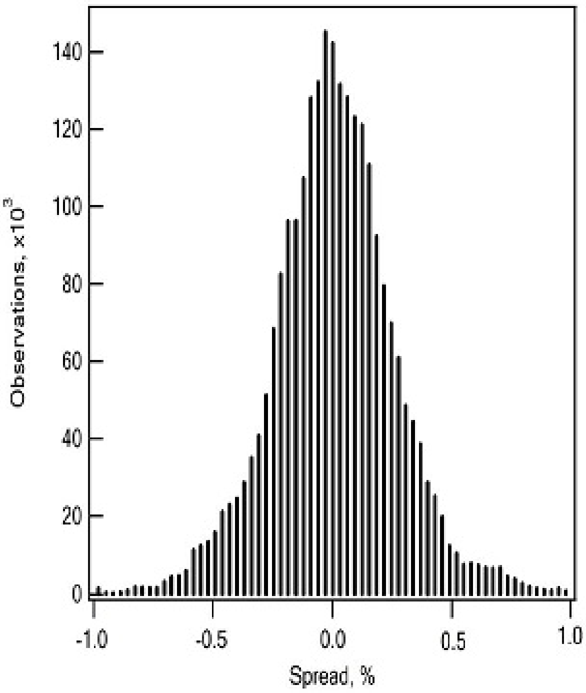

Figure 3: Empirical distribution of RDS-spread.

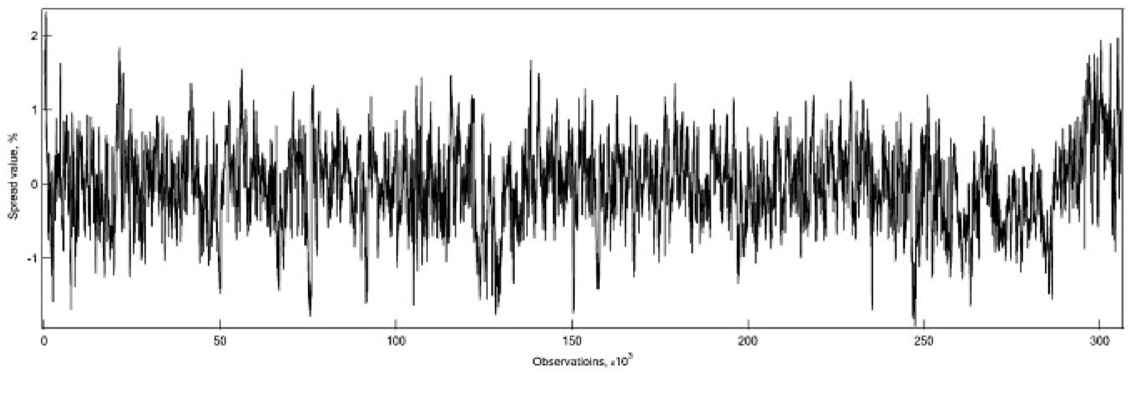

Figure 4: Log-price difference between ANZ.NZ and ANZ.AX.

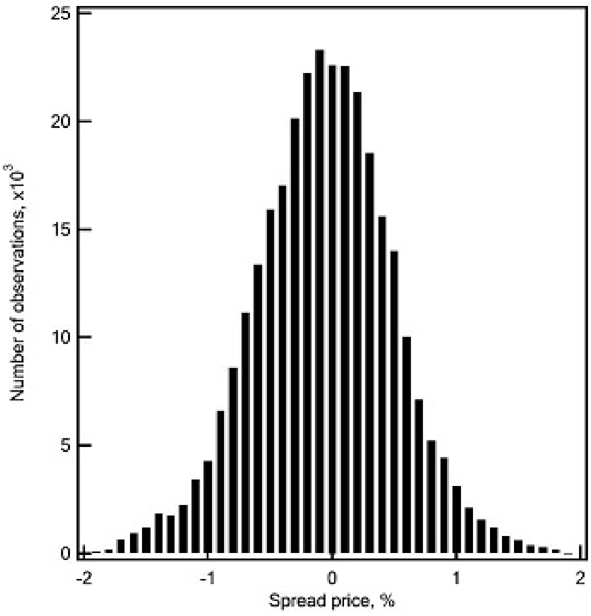

Figure 5: Empirical distribution of ANZ-spread.

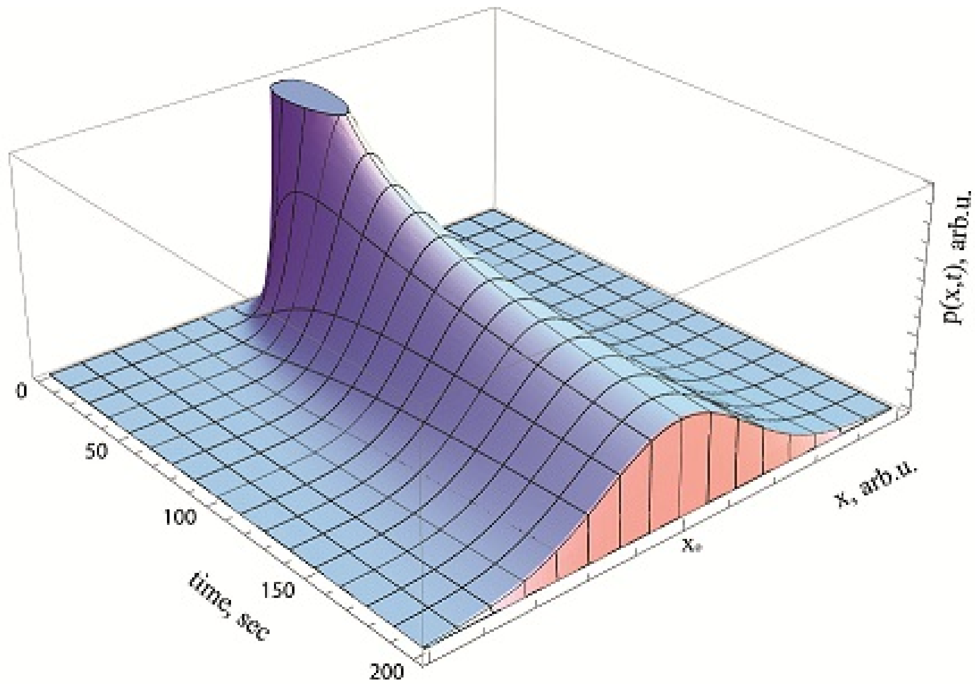

Figure 6: Modelled probability density function of finding a log-price x at time t, $p(x,t)$ for the Markovian master equation (8).

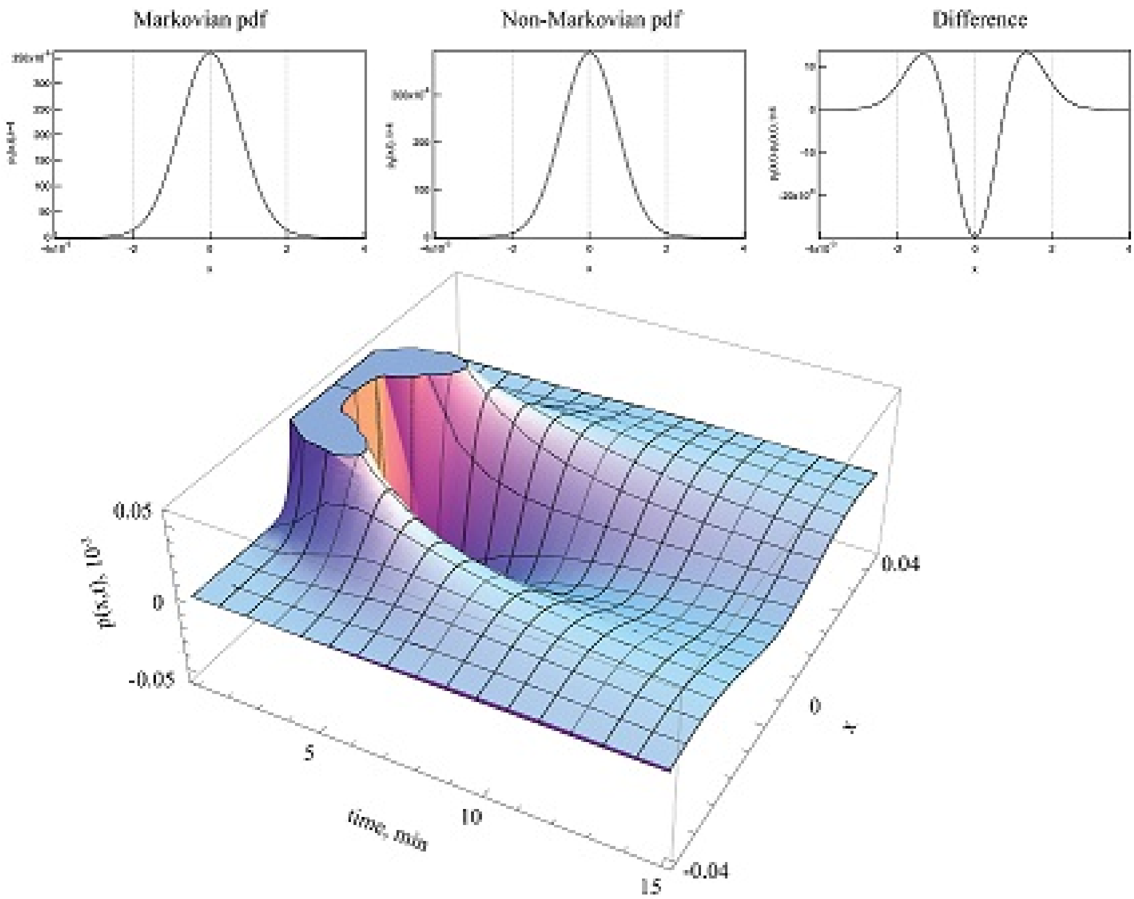

Figure 7: On top: The graph on the left illustrates the Markovian PDE, the central figure depicts the non-Markovian PDE. The cross-sectional difference between the non-Markovian and the Markovian PDEs at is displayed on the right. At the bottom: 3D plot displays the differences between the two PDEs over time.

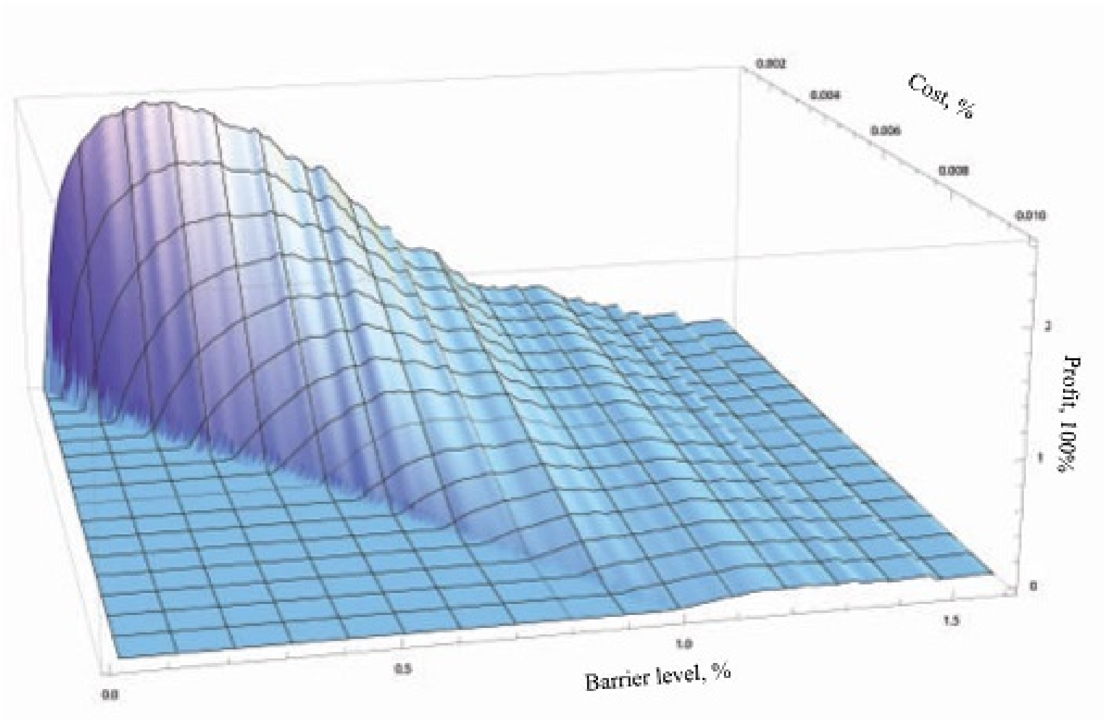

Figure 8: Profitability of the pair-trading ANZ-spread strategy. Profitability depends on transaction cost and barrier level.

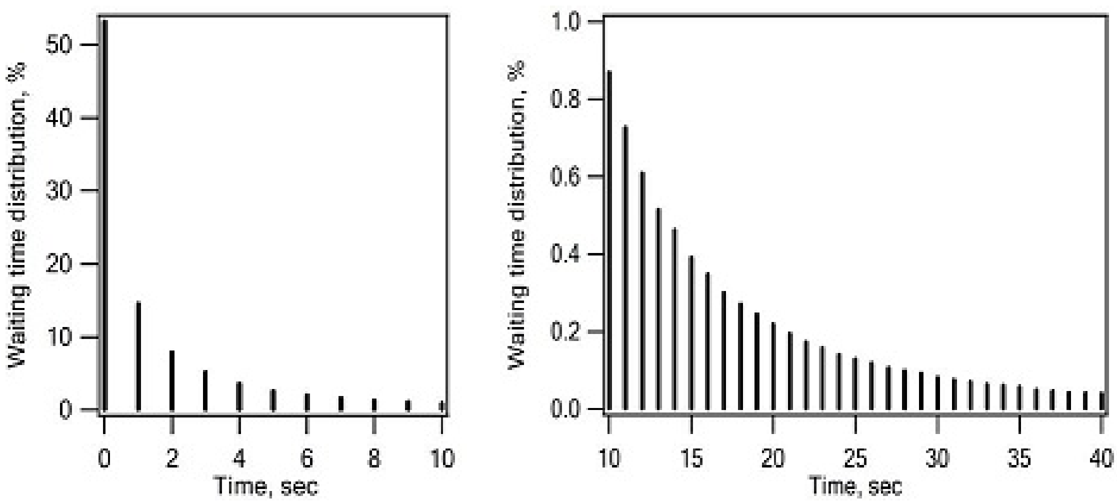

Figure 9: Waiting time distribution for RDS-spread.

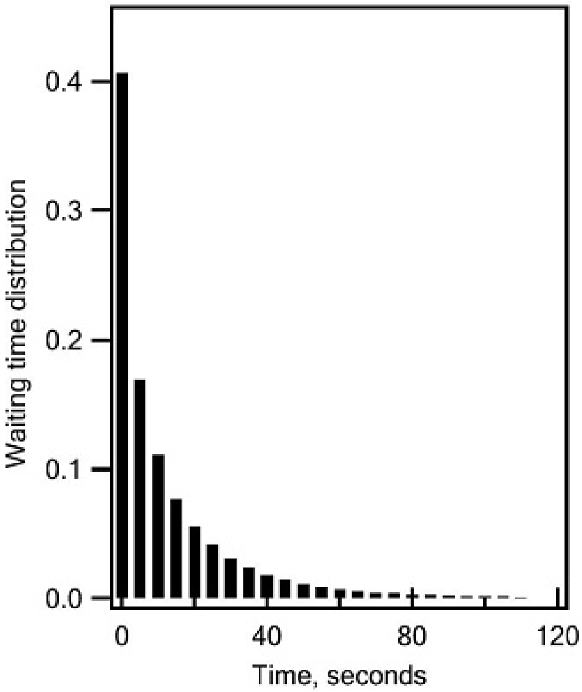

Figure 10: Waiting time distribution for ANZ-spread.

	90% inside	10% outside	1% outside
		RDS	
1 sec	52.6%	58.2%	58.0%
2 sec	67.1%	72.1%	73.3%
3 sec	75.0%	79.2%	81.1%
4 sec	80.2%	83.9%	86.0%
5 sec	83.8%	86.9%	89.1%
		ANZ	
5 sec	34.0%	35.8%	34.4%
10 sec	53.0%	55.4%	56.2%
15 sec	65.2%	67.7%	68.8%
20 sec	73.6%	75.9%	76.5%
25 sec	79.7%	81.7%	82.1%

Table 2: Probability of trade occurrence over time for the three cases: 90% time inside the barriers, 10% time outside the barriers, and 1% time outside the barriers.

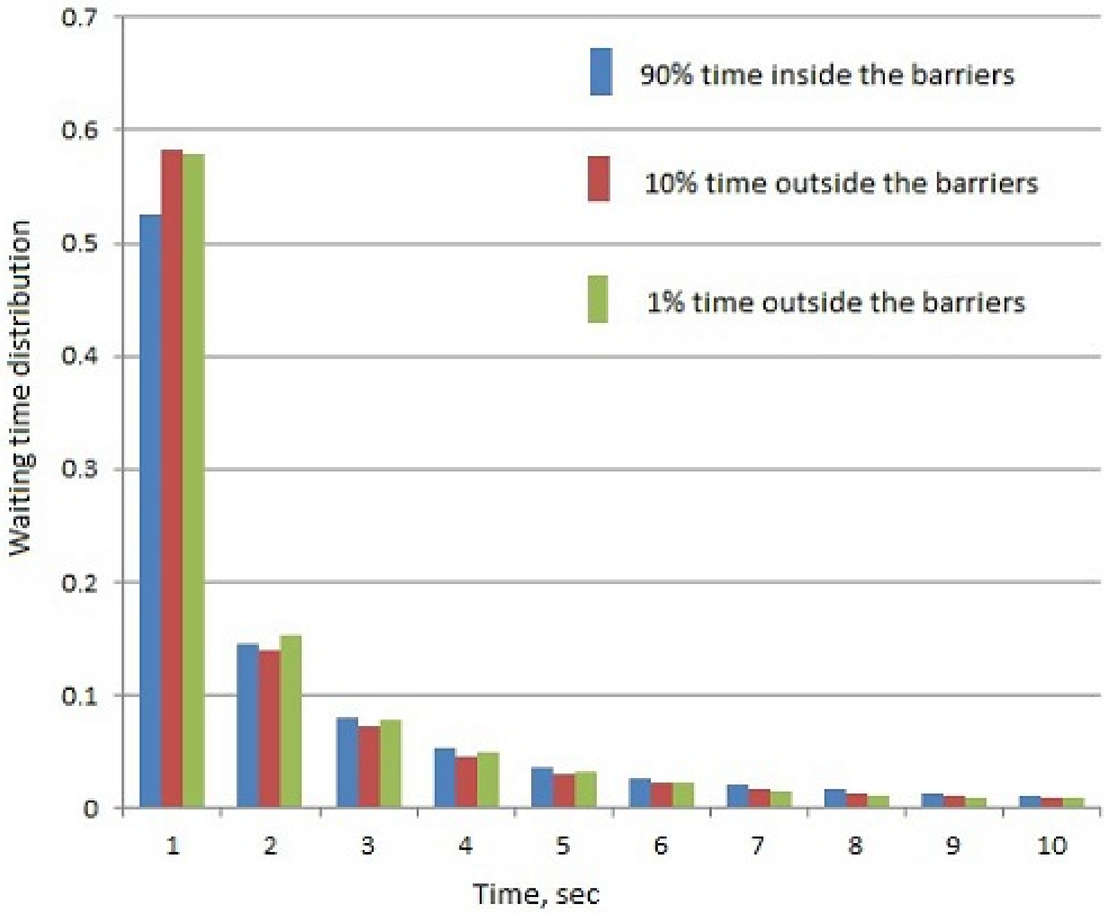

Figure 11: Waiting-time distribution as a function of the barrier levels for RDS-spread. Blue bars correspond to the barrier levels when the spread is 90% time inside the barriers. Red is for 10% time outside the barriers and green is 1% outside.

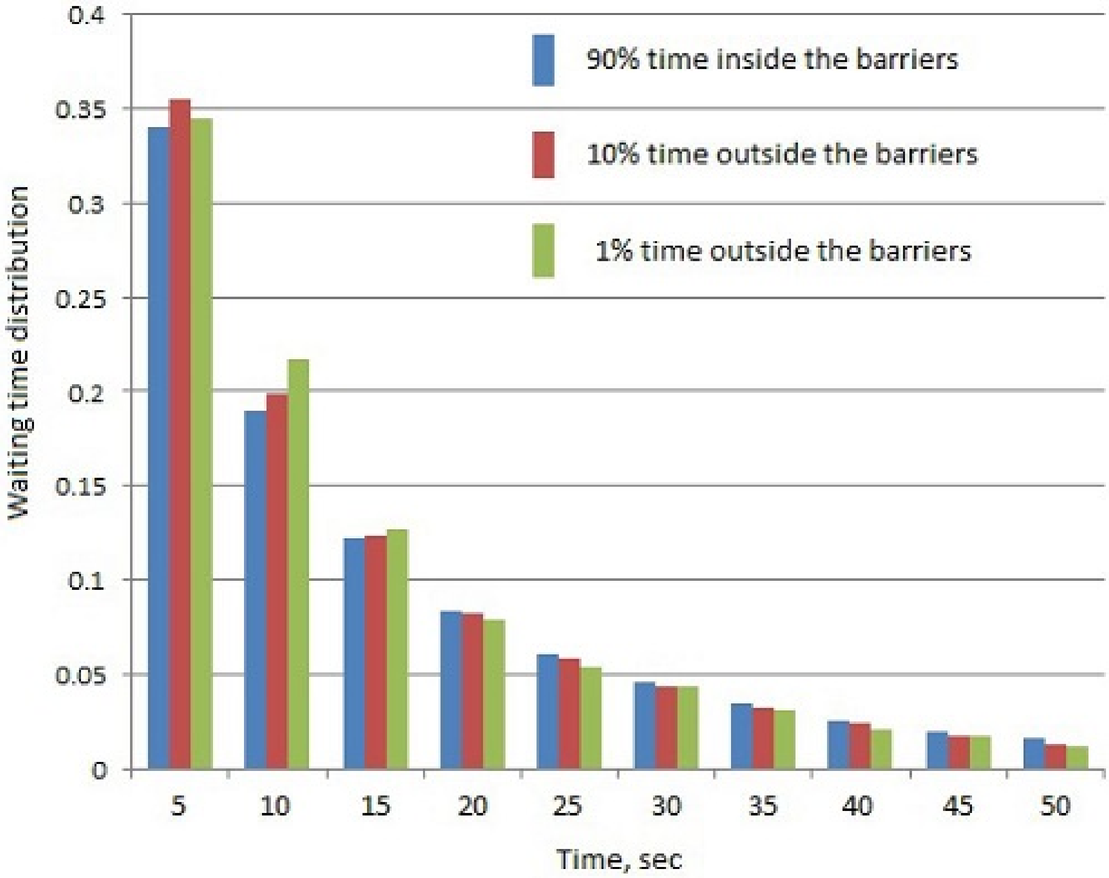

Figure 12: Waiting-time distribution as a function of the barrier levels for ANZ-spread. Blue bars correspond to the barrier levels when the spread is 90% time inside the barriers. Red is for 10% time outside the barriers and green is 1% outside.